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<table>
<thead>
<tr>
<th>WP Leader</th>
<th>Task Leaders</th>
<th>Document Type</th>
<th>Due date</th>
<th>Delivery date</th>
<th>Editor</th>
<th>Contributors</th>
<th>Version</th>
<th>Dissemination Level</th>
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<tbody>
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### D3.4: Potential for re-accumulation of hazardous large bodies from impact ejecta

**NEOShield**

A Global Approach to Near-Earth Object Impact Threat Mitigation

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<tbody>
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<tr>
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### Version control / History of Changes

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<th>Version</th>
<th>Author</th>
<th>Change description</th>
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1 Purpose of Study:

The objective of this study is to establish, through sophisticated simulations, regions of space safe for observing spacecraft in the vicinity of an asteroid after a kinetic impactor bombardment for specific observation time intervals.

2 Overview:

In order to build an informative model to assess probable locations of hazardous debris, we take the general approach to allow discrete pieces of ejecta to evolve in an $N$-Body code adapted specifically to this purpose. The $N$-Body code pkdgrav (Stadel, 2001; Richardson et al. 2000) is well suited to model discrete material in the sub-sonic regime; moreover, this code is able to compute the mutual self-gravities for a large number of particles in a time-efficient manner, which is necessary since the number of macroscopic ejecta from an impact can be very large ($>10^6$). Depending upon the particle density, typical collisional speeds, and other more nuanced factors, one of two different collisional routines that we have implemented in this code, namely the Soft Sphere Discrete Element Method (SSDEM; see Schwartz et al. 2012) or the Hard Sphere Discrete Element Method (HSDEM; see Richardson et al. 2011), both available to pkdgrav, may be more appropriate.

This study is primarily concerned with material lofted off the asteroid (target) surface that may “linger” in the asteroid’s vicinity for some time as a result of the bombardment. More difficult to assess than the dangerous high-speed particles that escape initially, this low- to moderate-speed material may also pose a threat to the operations of visiting spacecraft.

The overall objective is to generate a reasonably accurate ejecta state, suitable for our $N$-body code pkdgrav to then takeover and integrate forward in time through the time intervals of interest. In general, to simulate the actual impact, an appropriate hydrocode equipped with a model of material failure is what should be used in this regime since hypervelocity impacts include a fragmentation phase of impacted material. However, other means can also be used. Briefly, approaches to estimating the ejecta state just following the fragmentation phase are: (i) using what is known from cratering studies to analytically assign positions and velocities to granular material to match the essential features of an early stage of crater formation and ejecta evolution, (ii) simulating the initial impact using a hydrocode (in this case, an SPH code; Jutzi & Michel, 2014), and (iii) simulating the initial impact using pkdgrav (SSDEM collisional handling, with a particularly stiff equation of state). In the first case, analytical scaling laws are taken as inputs for the $N$-Body simulation computing the ejecta evolution using full gravity. In the second case, we use a numerical code capable of modeling the impact event, then, once the impact phase
is complete, we take the ejecta properties as inputs to our $N$-Body code, and compute the fate of ejecta using full gravity and soft-sphere collisions. And in the third case, the entire simulation is performed using the $N$-Body approach, with validation for the state of the (low- to moderate-speed) post-impact phase ejecta to be provided by scaling laws. These approaches are explored below.

3 Initial conditions and numerical methodologies

For this study, we considered the ejecta resulting from a 6-km/s artificial projectile impact into a body measuring 150-m across, suitable for the NEOShield design. In this section, we first explain our approach that uses experimentally derived scaling relations (Section 3.1) and our approach that uses SPH (Section 3.2) before showing how they are then used in an $N$-Body code to analyze the ejecta fate post-impact (Section 3.3). Lastly, we describe our approach to using $N$-Body simulation to calculate the fate of low-speed ejecta (Section 3.4).

3.1 Generating initial conditions using empirical scaling laws

Scaling laws to describe impact ejecta have been derived from experiments in Housen & Holsapple (2011), as well as Holsapple & Housen (2012), among other works. We have taken their scalings, using the parameters below, to create our baseline simulation.

Housen & Holsapple examined the data from experiments and assessed our understanding of how the ejecta velocity and mass distributions depend on the conditions of an impact event, in particular on the impact speed and target properties such as strength and porosity. To this end, they developed a point-source scaling model for the ejecta mass and velocity distributions, which fits the data for several classes of materials distinguished by porosity.

We used this scaling model to assign ejecta mass and velocity distributions, assuming a crater size of 27 meters across (of the same order as what is expected from these scaling laws and simulations). Our aim is not to make a precise prediction of the actual event with these initial conditions, but rather to get a sense of the number of particles (and their trajectories) that may cause a threat. Since the previous progress report, simulations using these conditions were performed, and we describe here our approach and define precisely those conditions. Results are then presented.

We start by using the following parameters/ constants for Eq. (14) in Housen & Holsapple (2011), which describes the velocity of material for a given distance from the point of impact. (These parameter values are close to their curve C5, shown in their Table 3.):

- Impact velocity, $U = 6$ km/s
- Crater’s radius, $R = 13.5$ m
- Constant, $C_1 = 0.55$
- Projectile’s radius, $a = 45.7$ cm
- Target’s bulk density, $\rho = 1.3$ g/cc
- Projectile’s bulk density, $\delta = 1$ g/cc
- Material constant, $\mu = 0.41$
- Material constant, $\nu = 0.4$

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Constant, $n_1 = 1.2$
Constant, $n_2 = 1.2$
Constant, $p = 3.3$.

For simplicity, we will assume a $45^\circ$ ejection angle for all ejecta.

The first step in this approach is to numerically simulate the filling of a “tub” with particles of size order 10 cm, and allow those particles to come to rest. The tub is defined as a spherical shell with radius of curvature $r_c$ and “opening angle” $\phi$. The procedure to simulate the filling of a tub with particles involves complex computations using pkdgrav and wall conditions that are described in-depth by Schwartz et al. (2012). We skip the details associated with this procedure and simply indicate that the outcome is an ensemble of particles being ejected with an angle of $45^\circ$ with ejection speeds decreasing with increasing distance from the impact point described by Eq. (14) of Housen & Holsapple (2011):

$$\frac{v}{U} = C_1 \left[ \frac{x}{a} \right]^{-1/\mu} \left( 1 - \frac{x}{n_2 R} \right)^p, \quad n_1 a \leq x \leq n_2 R. \quad (1)$$

(Note that we have adjusted the parameters as defined in the previous report in order to create more low-speed ejecta, as would be expected from a body whose surface is covered in regolith. Our first attempt, using $p = 0.3$ and $n_2 = 1.3$, produced no ejecta below the tiny escape speed of the body).

3.2 Generating initial conditions using smoothed-particle hydrodynamics

Below, we describe the conditions of the SPH simulation, how the results of the simulation are adapted to initial conditions for use in our $N$-Body code, and then discuss some of the problems encountered thus far with this technique.

3.2.1 Outputs from SPH simulations of the fragmentation phase

The initial conditions of the simulations of ejecta fate are given by the outputs of SPH simulations of the fragmentation phase explored by Jutzi & Michel (2014); these SPH simulations consider a hemispherical domain in the impact region of radius $\sim 17$ m.

Two SPH simulations have been used. One considers a bounded domain (particles confined within a finite hemispherical boundary)—we report on results from these simulations here; the other one, which is currently under investigation, considers a free domain (free particles in space).

The number of particles in this domain is about 4 million (4,262,972, exactly), which results in a particle mass of 3.15 kg and a particle radius of 0.0754 meters, assuming a target bulk density of 1.3 g/cc and a packing efficiency of about 74% (Hexagonal Close Packing). The projectile’s mass and density are 400 kg and 1 g/cc, respectively, and its impact direction is vertical along what is defined as the z-axis.

The simulations are run for 0.16 sec or 0.3 sec in the unbounded case and 0.16 sec in the bounded case. Although there does not seem to be high-sensitivity to the end-time of the simulations, investigations are currently being performed to better determine the most proper end-time of the SPH runs.

Once fragmentation is over, the output file has the following format to describe the properties of each SPH particle: id, bodynr, x, y, z, $v_x$, $v_y$, $v_z$, m, $\rho$, $h$; where:
id is the particle ID number,

\(bodynr = 1\) indicates target particle (N = 4262805)

\(bodynr = 2\) indicates projectile particle (N = 167)

\(x, y, z\) describe the particle position

\(v_x, v_y, v_z\) describe the particle velocity

\(m\) and \(\rho\) are the particle’s mass and density, respectively

\(h\) is the smoothing length (SPH parameter).

### 3.2.2 Porting SPH outputs into \texttt{pkdgrav}

The next step is to port SPH outputs into our \textit{N}-Body code, \texttt{pkdgrav}. To follow the gravitational evolution of large numbers of particles (up to several millions) and compute their gravitational interaction and potential reaccumulation (including the contact physics involved), an efficient \textit{N}-body code is required. We use the parallel \textit{N}-body hierarchical tree code \texttt{pkdgrav} (Stadel, 2001; Richardson et al. 2000) to compute the dynamics. The tree component of the code provides a convenient means of consolidating forces exerted by distant particles, reducing the computational cost. The parallel component divides the work evenly among available processors, adjusting the load each time step according to the amount of work done during the previous force calculation. The code uses a straightforward second-order leapfrog scheme for the integration and computes gravity moments from tree cells to hexadecapole order. Particles are considered to be finite-sized spheres and contacts are identified each step using a fast neighbor-search algorithm. The code was adapted to treat hard-sphere collisions for planetesimal modeling and asteroid disruptions/reaccumulations (Richardson et al. 2000; Michel et al. 2001) and later for granular material modeling thanks to the implementation of both HSDEM and SSDEM (Richardson et al. 2011, Schwartz et al. 2012).

In these simulations, the SPH particles each have identical mass, but their radii are less well defined. In effect, because the SPH technique treats material as a continuum, the particles do not have a physical radius but rather are characterized by a smoothing length (\(h\)) that connects them to the rest of the system. As a result, particles’ radii must be assigned. To do so, first a physical radius is derived from the mass and bulk density given in the SPH conditions, then steps are taken to ensure that these now discretized particles are of reasonable size and do not suffer overlaps (i.e., they are physically separated). This information needs to be transformed into a format adapted to the \textit{N}-body runs.

The file format of \texttt{pkdgrav}, describing the properties of each \textit{N}-Body particle, is the following: \texttt{pID, origID, m, r, x, y, z, v_x, v_y, v_z, \omega_x, \omega_y, \omega_z, c}; where:

- \texttt{pID} is the particle ID (ordered in increasing radius),
- \texttt{origID} is the particle’s original ID (from the SPH simulation),
- \(m\) and \(r\) are particle’s mass and radius, respectively (all are equal initially),
- \(x, y, z\) describe the particle position,
- \(v_x, v_y, v_z\) describe the particle velocity,
- \(\omega_x, \omega_y, \omega_z\) describe the particle spin frequencies (each set to zero initially as SPH particles do not have spin),
- \(c\) is a color index used for visualization.

Although the physical radii have been set to avoid too many overlaps, it is best to remove all instances of overlap at the start of the \textit{N}-Body simulation to avoid
unrealistic mutual repulsion of particle-pairs; thus we first run pkdgrav for one step, outputting all instances of remaining particle overlaps. For each particle that has overlaps, the greatest overlap is identified and the particle’s radius is shrunk by half this value (e.g., if a particle’s greatest overlap is 30% of its radius, the particle radius is shrunk to 85% of its original radius). Our approach entails that particles from areas of high density at the SPH-handoff remain high-density particles for the duration of the simulation. However, we have considered a technique to restore the appropriate densities when once particles leave these high-pressure regions.

3.2.3 Encountered difficulties using SPH

Although the procedure described above does not seem to suffer from major issues, there are in fact some real difficulties in generating the needed data for the study of the gravitational phase of the ejecta from SPH simulations.

In particular, the main interest of industries for the design of an observing spacecraft concerns these low-speed ejecta, i.e., those that are ejected at or near the escape speed of the target. They may not immediately escape the system, but rather go into orbit or stay around for some time before escaping or falling back onto the asteroid’s surface. Given the low gravity, and therefore the low escape speed, of considered targets (of the order of cm/s) as well as the high impact velocity (> 6 km/s, at least five orders of magnitude greater than the escape speed), the computation of the velocities of ejecta get close to the numerical noise of the SPH simulations. Therefore, it is difficult to assess the reliability of the low-speed ejecta output. Moreover, although in the SPH simulations the impact region is initially rigid (strength dominated), in reality, loose particles (regolith) may exist at the surface of an asteroid (even a small-sized one) and the impulse required to loft those particles (provided that they are not stuck by strong cohesive forces) is very small, and this is not accounted for in SPH runs. Although we provide results starting from SPH-simulation output (ignoring for the most part the potential presence and contribution of regolith), assuming that the low-speed ejecta are well accounted for (see Section 4), we have also looked into other ways to investigate the ejecta fate. In future works, building from our experience and difficulties encountered with these SPH simulations, we plan to perform new SPH simulations using other boundary and surface conditions, pushing the limits of existing software, so that we can access the low-velocity ejecta with greater reliability.

3.3 $N$-body simulations (post-hypervelocity impact phase)

Once the particles’ post-hypervelocity impact phase positions and ejection velocities are set, either from interpreting the output from SPH runs (Section 3.2) or from the direct assignment of velocities from scaling laws (Section 3.1), the $N$-body simulations to determine the ejecta fate can be performed. However, the environment of the asteroid remains to first be defined. For this, we use a spherical wall (see Schwartz et al. 2012) as an approximation of the boundary (surface) of the asteroid (see Fig. 1). In order to account for the full gravity of the asteroid, we affix a particle in the center of the body, “stuck” to the center of the spherical wall with the mass of the entire asteroid. The radius of this particle is not important so long as it is sufficiently inside the radius of the asteroid (i.e., the spherical wall), such that it suffers no collisions. We happen to use 1 cm for the radius of this particle serving as a proxy for the asteroid mass.
All particles from the end of the SPH impact phase are then positioned such that those closest to the surface remain just above the surface of spherical wall (Fig. 1). The placement above the wall is to ensure that there are no particles overlapping with the wall (which would entail unrealistic repulsive forces), while still keeping them close to the surface, maintaining the accuracy of the gravity calculations. In the case where we used scaling relations, we were able to preserve the case of a perfect sphere for this baseline simulation by carving out a section of the spherical wall and placing a bowl inside it filled with particles such that the rims of the bowl and of the carved-out section aligned. We also heaped particles on top of the bowl such that the surface of the bulk of particles rounded out the sphere of the 75-m radius body (Fig. 2; top).

Figure 1: Initial positions of particles after the hypervelocity impact phase for one of the ejecta-evolution simulations involving a 1.5-km-asteroid target. The large sphere below the ejecta material represents the asteroid surface.

For the SPH-derived initial conditions, since during their ejections, particles may interact physically with each other, material parameters are defined to account for the various contact forces/frictions (e.g., static, rolling, sliding, and twisting frictions). For the case using scaling relations, we are in the process of including collisional interactions between particles. Indeed, the expected number of interactions between particles is rather small relative to the total number of particles, as most are on ballistic trajectories and the system is not a particularly dense one. However, collisions could play some role in changing the orbital trajectories of a small fraction of particles, possibly keeping them in the vicinity for more time.

For this study, we are not interested in the fate of particles that we know will eventually land and remain on the surface. Therefore, as the simulations progress, if
a particle impacts the spherical shell (asteroid's surface), we typically make it vanish (delete it from the simulation) when it impacts the shell, which has the advantage of decreasing the number of particles along the way.

For the SPH-derived initial conditions, the procedure to start the next (N-Body) phase of simulations, even after the post-hypervelocity-impact-phase particles are successfully ported into pkdgrav, is not trivial and involves several steps (the procedure based on scaling relations without collisions is less sensitive to the choice of timestep). The first step (for SPH-derived initial conditions) is to perform a portion of the overall simulation using a very small timestep (1 ms) and integrate the system over 100 s (i.e. $10^5$ steps). The reasoning here is that because particles are initially moving very fast with very large relative speeds, the stiffness of the spheres has to be large enough to accommodate those relative speeds when they collide, and as per Eq. (4) in Schwartz et al. (2012), the higher stiffness parameter necessitates a smaller timestep. Then, after 100 s, when particles are more dispersed in space and collisions tend to take place occur at lower relative velocities, the stiffness parameter is decreased by a factor of 100, allowing the timestep to be increased to 10 ms. The system is integrated further until 1 ks (90 more ksteps). We continue in this manner, progressively decreasing the particles’ stiffness while increasing the timestep with the goal to get to a point where the timestep is large enough such that the system can be evaluated weeks or months after the impact. Based upon the speed of simulations thus far carried out, we consider that this happens once a timestep of about 1 sec is reached. This corresponds to a stiffness parameter for very soft particles, where only relatively low-speed collisions can be accurately handled.

All throughout the simulation process, irrespective of the initial conditions chosen, we have various quality controls in place. These include warnings for large particle overlaps, an overall assessment of how many particles are in contact and to what degree of overlap, internal consistency checks (tracking neighbor/contact lists), etc. There are also energy, momentum, and even mass conservation issues to keep track of. For example, particle-particle collisions cause energy loss (due to friction); we do not account for the reaction force on the asteroid when particles collide with it (infinite inertia approximation); also, particles may stick or be deleted upon contact with the asteroid surface. Moreover, we need to note that many SPH material properties that are specific to the SPH phase, such as pressure and material cohesion, are neglected in the N-body phase.

### 3.4 Using pkdgrav for the entire simulation

The final approach that we include here is to use an N-Body approach to simulate the entire process from start to finish. The difference between this and what was described in Section 3.3 is that we attempt to use pkdgrav with SSDEM to account for the impact/fragmentation phase—at least for the low-speed particles. This approach has been used to reproduce laboratory experiments involving low-speed impacts into granular material (regolith) in the context of JAXA’s Hayabusa-2 mission (Schwartz et al. 2014). Although SSDEM is not strictly appropriate for modeling the fragmentation and energy propagation that ensues from a hypervelocity impact, the intent here is to capture the effects of low-speed regolith that may loft off of the asteroid surface several meters away from the impact site, material poorly accounted for with SPH. Using this approach for the entire integration, from impact through the end of the ejecta evolution phase, would allow for a numerical simulation.
of the low-speed ejecta resulting from the impact while avoiding the drawbacks of using SPH (touched upon in Section 3.2.3), including the difficulties involved in switching between different coding platforms (Section 3.2.2).

![Figure 2](image)

Figure 2: The start (top frames) of an N-body simulation using ejecta particle velocities solved for with scaling laws from Housen & Holsapple (2011): a bowl of 641,586 particles representing a portion of the surface of an asteroid that suffers a kinetic impactor strike is embedded into the surface of a sphere that represents the entire asteroid; the particles are then ejected (bottom frame). A small particle is fixed to the center of the body with a mass that represents the mass of the entire asteroid of bulk density 1,300 kg/m$^3$. Particles evolve in accord with their mutual gravities.

### 3.4.1 Logistical consideration: Computational expense

This subsection of the report works through toward an estimate of how much computational time such an SSDEM simulation would require. Solving for a hypervelocity impact and the ensuing few seconds using SSDEM requires the use of particles with relatively high $k_n$ (a stiff equation of state) in order to ensure that the interpenetration between particles does not get too large (e.g., the maximum penetration, $x_{\text{max}}$, should certainly be less than a particle’s radius), where
The first term in the bracket represents a kinetic constraint (due to collisions) on the strength of the elastic repulsive force, while the second term represents a static constraint (due to confining pressure) on the strength of the elastic repulsive force. Here, $\Sigma$ would be some estimate of the column density atop a particle of radius $s$, $a_g$ is a uniform acceleration due to gravity acting on the column supported by the particle, $\mu$ is the reduced mass of the system, and $v_{\text{max}}$ is the maximum expected relative speed between particles. The value of $k_n$ in turn, determines the choice of timestep, which would be of the order

$$
\Delta t \sim \pi \frac{\mu}{n_{\text{overlap}} \sqrt{k_n(1-\xi^2)}} \tag{3}
$$

where $n_{\text{overlap}}$ is the number of steps taken to resolve the half-oscillation of a spring with frequency given by the square root term (a conservative choice would be $n_{\text{overlap}} = 50$), and where $\xi$ is given by the coefficient of normal restitution $\varepsilon_n$ as

$$
\xi \equiv \frac{\ln(\varepsilon_n)}{\sqrt{\pi^2+\ln^2(\varepsilon_n)}} \tag{4}
$$

Combining (2) & (3), approximating $1/(1-\xi^2)$ as unity, and taking stiffness to be determined by the kinetic constraint, this gives

$$
\Delta t \sim \frac{\pi}{n_{\text{overlap}} v_{\text{max}}} x_{\text{max}} \tag{5}
$$

To estimate the timestep that might be appropriate to simulate this impact, let’s take $v_{\text{max}} = 6 \text{ km/s}$ and $x_{\text{max}} = 20 \text{ cm}$ (the maximum overlap occurs just at the initial point of impact, but must still be less than the radius of the smallest particle in the system). This gives a conservative estimate for the timestep of about 4 $\mu$s (numerical trials for this specific simulation show the ability to use a slightly softer equation of state and up to a 16 $\mu$s timestep). Using a single-node AMD Opteron 6284-SE machine (32 cores of 2.7 GHz), advancing by one step a self-gravitating 450,000-particle simulation takes about 3 seconds, although this time could be shortened (for example, this estimate was reached while checking each particle for contacts with its 32 closest neighbors; this number may be decreased depending on the size ratios of particles, shortening the integration time. The number of particles in the simulation can also be decreased.). Nevertheless, this provides a basis from which to make some estimates. Thus to simulate one second using this setup takes just over a week (or 2 days, using a timestep of 16 $\mu$s). But our task is to assess the fate of ejecta for several hours and even days after impact. To perform a simulation on a computer node such as this one, starting from impact up through ejecta evolution, would require at least a thousand-fold increase in integration speed to be complete within a few weeks. We have fortunately come up with a solution, presented below.
3.4.2 Mitigation technique: discrete particle layers

Collisional energies that take place over the course of these simulations range enormously at different epochs of the simulation. There is room to exploit, for computational efficiency, the fact that high-energy collisions occur at and immediately following the initial impact, but do not continue to later times. One solution is to change the stiffness, the $k_n$, of the particles after the impact phase (seconds), once relative speeds of colliding particles decrease significantly. This allows for a switch to a larger timestep at this point, with $h \propto k_n^{-1/2}$. However, decreasing $k_n$ reduces the energy contained in existing contacts between particles $\frac{k_n s^2}{2}$, and material still slowly excavating from beneath the surface would be affected by this loss of energy. Instead, we have come up with a solution involves the use of heterogeneous particles, specifically particles with two discrete layers, each layer having the ability to be assigned its own material properties.

What is needed is to have a very stiff equation of state that acts only during the first phase of the simulation when collisional speeds are high. To carry this out, particles with a very stiff inner core (high $k_n$) and a pliant outer layer of material (low $k_n$) are used. The particles’ stiff cores play an important role during the first phase of the impact, but are not involved during the second phase, allowing for an increase in the integration timestep. This relies upon the assumption that high-energy collisions cease. It is also worth reiterating that we are not trying to explicitly model a specific material, but rather trying to determine the fate of relatively low-energy, regolith-like ejecta. Here, we present an implementation of a technique to surround each particle with a softer surface layer.

The pliant outer layer extends to a depth $s_{outer}$ beneath the surface of each particle, with $s_{inner}$ being the radius of the stiff inner core, where $s_{inner} + s_{outer} = s$, the (entire) radius of the particle. The idea is to begin the simulation using a timestep appropriate for contacts that involve the inner core, but then once particles no longer suffer high-energy collisions, the timestep can be increased to a value appropriate for contacts that involve only the pliant outer layer. First, appropriate values for the two types of materials must be chosen, along with an appropriate position for the boundary layer between them.

When $x > s_{outer}$, the normal repulsive force has a magnitude equal to

$$k_{n,outer} s_{outer} + k_{n,inner} (x - s_{outer}).$$

(6)

In order to choose an appropriate timestep for the outer layer, we use analysis like what was included in Section 3.6.1. We skip these calculations and report that the kinetic constraint (rather than the pressure constraint) determines the stiffness of this layer, and choose a stiffness that meets two criteria: it ensures that collisions of a few times the escape speed of the body can be supported, and it allows penetration into the inner layer to cease within about 10 seconds of the impact when using an outer layer of 10% of the radius. This allows the timestep for when overlaps only occur within the outer layer to be 50 ms, which, although still computationally expensive, allows a simulation of hundreds of thousands of particles to be complete within an acceptable amount of CPU time.
4 Results

We have explored each of the methodologies described in Section 3 and provide the results to simulations performed thus far. Further applications are ongoing and will be explored in Section 5. For each of the three methods described in Section 3, we present results of simulations of ejecta fate based upon these methods, each of which we continue to develop. [In the case of the full N-Body simulation (Section 3.4), we show results only for the first phase of the simulation for reasons explained below.] We have developed helpful visualization tools, which we include here as applied to these first ejecta-fate-analysis cases. These results are visualized in Figs. 4–6 (scaling relations), Figs. 7–9 (SPH-derived initial conditions; unbound domain), and Figs. 10–12 (pkdgrav only). The first figure in each of the three sets (Figs. 4, 7, and 10) show three cross-sections above the surface of the body through averaged-density space; that is, masses contained in cubic volumes of space (mesh-sizes are specified in figure) are smoothed over these cubic regions. Integrating these densities over specified time intervals correlates to the chances of finding ejecta in these regions, thus providing a quantitative sense of the relative safety of these regions. The second figure in each of these sets (Figs. 5, 8, and 11) show smoothed energy densities in the same way that the first set of figures show smoothed mass densities. The third figure in each of these sets (Figs. 6, 9, and 12) show the mass densities for different cross-sections, and given as fractions of the total ejecta. These visualization techniques (along with others being developed) will be needed for the range of less idealized scenarios (including specific cases of actual NEOs) that will be studied as part of WP11 in NEOShield-2.

4.1 Simulation results with initial conditions derived from scaling relations

Our baseline simulation, the most straightforward to implement, was based upon assigning velocities to particles using experimentally derived scaling relationships (Section 3.1; Fig. 2). One result of the simulation is that particles close to the impact site escape quickly to infinity with no significant deviation in their flight path, while slower moving material closer to the nominal rim of the crater are deflected by the gravity of the body. The slowest moving material, which does not make excursions far from the surface, trace close to parabolic trajectories before re-impacting the surface. Between the extremes in ejection speeds, particles cover all spatial locations shallow to 45° from the surface, but at different times. Spatial volume elements follow a general trend that as distance from the body and angle from surface tangent increase, particle speeds increase while the mass density of particles decrease. Slowly escaping material is affected, not only by the gravity of the body from which it is escaping, but also by the mutual self-gravity of the ejecta (for the baseline simulation, collisions and solar perturbations are not in effect, but mutual self-gravity between all particles is taken into account). This “gently lofted” material has time to form under-dense and over-dense regions due to the gravitational forces; this can be seen in Fig. 2 (bottom image). We expect this effect to be exaggerated by the inclusion of dissipative collisions, one of the many additions being built atop this baseline model that will be included in WP11 of NEOShield-2, “Structural Modeling and Numerical Simulations.”
4.2 Simulation results with initial conditions derived from SPH-simulations

In contrast, the results using SPH-generated initial conditions show the initial, high-speed ejecta escaping at low angles of inclination, with the angle increasing for ejecta that escape at lower speeds. Fig. 3 shows the location of the ejecta as time advances (bounded domain). The top row contains ray-traced images of particle locations just above (within ~25 m of) the surface at four different epochs, while the bottom row contains plots of the azimuthally averaged positions of ejecta at these same epochs, but showing a region of ~6 km. We can see (from the bottom row of Fig. 3) that at 1 ks, ejecta at inclinations below 60° have been cleared from the 6 km vicinity. At later times this region repopulates, albeit sparsely, with slower ejecta due to gravitational, and possibly and collisional effects. By 10 ks (~3 hrs), the ejecta cone within a region of 6 km above the surface has been filled in by slow-moving ejecta (the ejecta is slow-moving relative to the high-speed ejecta, but it should be kept in mind that this ejecta is still at speeds well above the escape speed of the body). The discrepancies between the results of this methodology and the one based upon assigning initial velocities by scaling relations will be discussed in Section 5.

4.3 Results of initial phase of simulation carried out with N-Body code

A simulation using the method described in Section 3.4 has been completed, although we found that, when carrying out our specific simulation through to completion, significant boundary effects were seen at later times. However, we can report here that the methodology works just as described. After integrating the simulation to a time ~10 seconds post-impact (taking 11.5 days using \( N = 186,184 \)), particles ceased to penetrate beyond 10% of their radii—the radii of their outer surfaces—and thus the timestep was successfully increased by a factor of 3,000. In part to explore our visualization techniques for cases when the ejecta patterns are less regular, we include here the results of the initial phase of the pkdgrav impact simulation using particles without the softer surface layer (Figs. 10–12).

Figure 3: Positions of individual ejecta grains after a simulated hypervelocity impact on a 150-m (bounded domain) target (notice the curvature of the target compared with that of the 1.5-km target of Fig. 1). Top row: ray-traced images near the impact point; the images are centered on the point of impact. Bottom row: corresponding polar plots showing particle positions (red points) with concentric circles every 1 km of simulated space; azimuthal positions are not shown and the plots are centered on the asteroid center. In all images, the
impact trajectory is downward (z-axis points up). Snapshots progress in time from left to right, with the first frame corresponding to the SPH/pkdgrav handoff 0.16 s post-impact; the second frame is taken at 100 s; the third at 1 ks; and the final frame depicted here at 10 ks.

5 Analyses and perspectives

In this report, we have presented our developing approaches and the results they are bringing, to characterize the vicinity around a small body after an artificial projectile impact. The test case investigated here consists of a non-rotating, spherical body of uniform bulk density 1.3 g/cm$^3$, measuring 150 m across, and impacted normal to the surface at 6 km/s. These approaches all involve the numerical simulation of ejecta using an N-Body code to calculate their evolution using mutual self-gravity. Each of the methodologies includes detailed procedures that determine the initial placements of particles and the arranging the gravitational environment. Where they differ is in how the initial conditions of the ejecta are generated. The models at the moment do not incorporate the effects of solar radiation or those of solar (or planetary) tides, although we are in the process of adding in these effects.

One method involves assigning velocities to particles based upon experimentally derived scaling relations. For the simplified case of a spherical body investigated here, we have calculated the ejecta fate out several months, based on a reasonable set of parameter choices, and the assumption of 45° initial trajectories. This specific method did not include collisional interactions, while all other models have collisions included. Also, all ejected particles are assigned their initial velocities at the start of the simulation, approximating the excavation time to be instantaneous (this can be mitigated if we determine that it is important to determining the ejecta fate).

A second method involves taking the outcome of an SPH simulation as the initial conditions for the N-Body calculation. These two approaches have produced results that were quite different from each other, primarily in their angles of ejecta. Although it is appropriate for the development of a baseline model, it is clear that assigning a uniform ejection angle of 45° is too simplistic. And in the case of SPH, it is also unclear how realistic the result is of slow-moving ejecta lofting off the surface nearly vertically. Although it does seem to be the result of the chosen domain and boundary conditions during the SPH simulation (using an unbound domain during the SPH simulation still shows an ejecta cone narrowing in time, although to less of an extent than in the bound case), it is also the result of being in a microgravity environment. It could very well be that in reality much of this low-speed material lofting off the surface should be compacted into void spaces, wedged into configurations that would not allow them to escape as individual particles. However, we don’t really have reliable experiments that tell us what to expect from loose regolith in microgravity environments, and at least some lofting cannot be ruled out. We will investigate this further by experimenting with the size of the SPH domain and

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1 The first large-scale, active impact experiment on a small body was performed on July 4, 2005, by the NASA’s Deep Impact mission (A’Hearn et al. 2007), which targeted Comet 9P/Tempel 1. However, Holsapple and Housen (2007) reported that conventional cratering might not explain fully the observed plume of the Deep Impact event (acceleration mechanisms such as vaporizing ices most likely moved the material more quickly).
the boundary conditions of the domain. Since the methodology in Section 3.2 is adaptable to most other software platforms, we are also in the process of using multiple different classes of hydrocodes including different SPH codes to examine this and other ejecta behavior (and, the methodology in Section 3.1 is adaptable to other scaling relations as well). We will also incorporate more realistic ejection angles from scaling relations (still, there is only so much scaling relations can provide about this low-speed regime).

Related to the high-inclination ejection angles, since the previous report (August, 2014), the physical explanation for the filamentary strings seen to develop in the SPH-derived ejecta has been determined to be highly affected by the tidal forces of the body. Energy-dissipating collisions and/or inter-particle gravity may also help by playing a role, but the effect nearly disappears as the size of the body is increased.

Although we don’t have worthwhile results to present here for the strategy that involves only the use of the \textit{N}-Body code, we do report having successfully implemented the methodology. What was needed was a way to raise the timestep by many orders of magnitude so that a simulation could be completed in a reasonable amount of time. This involved having a very stiff equation of state that acts only during the first phase of the simulation when collisional speeds are high.

We continue to pursue this complete \textit{N}-Body approach because there is to date no good experimental predictions for particles moving at speeds of order centimeters-per-second for long periods of time under microgravity conditions. Using $p = 0.3$ and $n_2 = 1.3$, as suggested in Housen & Holsapple (2011) produced no ejecta below the escape speed. Although, their scaling laws do not extend down to anywhere near these speeds, we needed to alter these parameters (Section 3.1) in order to produce low-speed ejecta. Since our choice of parameters do not have experimental justification, we believe that solving for the very low-speed regolith with a granular code is the correct course to take. We are currently working to compare the ejecta to the scaling relations at speeds where both \textit{N}-Body ejecta distributions and scaling relations are justified. We can then, in effect, extend the scaling relations to the distributions of low-speed ejecta, essential for this study on ejecta fate.

5.1 Further ongoing improvements (in addition to those just mentioned)

- We will be including collisional effects into the simulations that use scaling relations to assign velocities; we have just recently calculated an appropriate range of particle stiffness to use in these simulations.

- Thus far with the \textit{N}-Body-only approach, we have simulated cohesionless regolith, and it might be that some cohesion is called for (we have successfully demonstrated the ability to add cohesion for sintered glass bead agglomerates; see Schwartz et al. 2013).

- So far we have not seen a need to use HSDEM in place of SSDEM at the late-stage ejecta phases, however, since HSDEM can be integrated faster in situations where collisions are rare, we continue to keep this option in mind.
• We are working on ways to adapt the setup from the simplified case studied here to the much more complex case of the Didymos binary system, the target of the proposed AIDA NEO-deflection mission. We have a strategy, based on what we have performed for the case of a deflection study on the NEO Itokawa (D9.6). For that case, we adapted the approach in Section 3.1 to the case of the surface of a rotating asteroid for which we have a shape model. Something similar will be done in the case of Didymos. Although the case of the binary makes it more of a challenge, we can take advantage of the assumption that the secondary is tidally locked to the primary.

• As mentioned in the opening paragraph to this section, the effects of solar radiation and solar tides will be added to the simulations. Planetary tides will also be added as needed.

• The technique in Section 3.2.2 involves shrinking SPH-particle radii such that there are no overlaps at the start of the N-Body simulation. In particularly dense regions, this can lead to the system containing a small amount of unrealistically dense ejecta. We don't see this as a major concern because the most significant effect of having dense particles is that they have smaller cross-sections, and collisions far from the body are rare. Nevertheless, we have developed a strategy (yet to be implemented) to allow these “too dense” particles to slowly expand in radius in a way that conserves angular momentum.

The predicting of trajectories of material ejected as a result of impact deflections or deflection tests is a complex numerical project. Such an undertaking requires detailed numerical approaches and significant computational time. At this point, we have developed sophisticated strategies to simulate the fate of the ejecta, which involves the use of a high-performance N-Body integrator, pkdgrav. Putting together what we learn from these different approaches, we have the ability to generate—given a range of initial conditions and assumptions—a reasonable assessment of where hazardous material may be and when.
References


Figure 4: Orthogonal-slice planes through volumetric mass density distribution (kg/m³). Mesh width = 0.33 km; slices at heights 1.2 km, 2.1 km, and 3.0 km. Based on Scaling Law initialized simulation (Section 3.1); nonlinear scale is used for the color bar.

Figure 5: Orthogonal-slice planes through volumetric (kinetic) energy density distribution (J/m³). Mesh width = 0.33 km; slices at heights 1.2 km, 2.1 km, and 3.0 km. Based on Scaling Law initialized simulation (Section 3.1).
Figure 6: Proportional distribution of the ejecta based on Scaling Law initialized simulation (described in Section 3.1). Mesh width = $2.5 \times 10^3$ km; nonlinear scale is used for the color bar.

Figure 7: Orthogonal slice-planes through volumetric mass density distribution (kg/m$^3$). Mesh width = 0.167 km; slices at heights 0.1 km, 1.0 km, and 2.0 km. Initial conditions derived from SPH simulation using unbound domain during impact (Section 3.2); nonlinear scale is used for the color bar.
Figure 8: Orthogonal slice-planes through volumetric (kinetic) energy density distribution (J/m³). Mesh width = 0.167 km; slices at heights 0.1 km, 1.0 km, and 2.0 km. Initial conditions derived from SPH simulation using unbound domain during impact (Section 3.2).

Figure 9: Proportional distribution of the ejecta based on initial conditions derived from SPH simulation using unbound domain during impact (Section 3.2). Mesh width = 1.0 km; nonlinear scale is used for the color bar.
Final Deliverable 3.4: Potential for re-accumulation of hazardous large bodies from impact ejecta (WP3)

Figure 10: Orthogonal slice-planes through volumetric mass density distribution (kg/m³). Mesh width = 5.0 m; slices at heights 80 m, 130 m, and 180 m. Initial phase of pkdgrav simulation (Section 3.4); nonlinear scale is used for the color bar.

Figure 11: Orthogonal slice-planes through volumetric (kinetic) energy density distribution (J/m³). Mesh width = 5.0 m; slices at heights 80 m, 130 m, and 180 m. Initial phase of pkdgrav simulation (Section 3.4); nonlinear scale is used for the color bar.
Figure 12: Proportional distribution of the ejecta based on initial phase of \textit{pkdgrav} simulation (Section 3.4). Mesh width = 25 m; nonlinear scale is used for the color bar.